

Inflatonless inflation

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Abstract

Whichever could be the real theory of gravitation, the corresponding low-energy effective lagrangian will probably contain higher derivative terms. In this work we study the general conditions on those terms in order to produce enough inflation to solve some of the problems of the standard Friedmann-Robertson-Walker cosmology in absence of any inflaton field. We apply our results to some particular scenarios where higher derivative terms appear in the effective lagrangian for gravity like those coming from graviton (two)-loops or integrating out ordinary matter (like the one present in the Standard Model) .

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1 Introduction

In the last years, the general physical idea of a period of enormous inflation in the early history of our universe has emerged as one of the most simple and successful ways to solve many of the problems of the Friedmann-Robertson-Walker cosmological scenario [1,2]. The list of those problems includes flatness, the horizon problem and the origin of the density fluctuations needed to produce the current galactic structure.

Concerning to the nature of the physical mechanism responsible for the inflation there are many of them proposed in the literature. However no one has universal acceptance. Typically they include the action of some scalar field called inflaton taking some vacuum expectation value. The corresponding energy density plays the role of an effective cosmological constant which gives rise to a de Sitter phase of exponential expansion. The concrete nature of the inflaton field depends on the different microphysics models considered.

In this paper we deal with another kind of approach based on the possibility of having inflation without any inflaton field. This possibility was envisaged by Starobinsky some time ago [3]. Even before the inflationary paradigm was established, this author discovered that the addition of some terms to the Einstein equation of motion gives rise to de Sitter spaces as solutions. In fact those terms can be obtained in the effective action for gravity as the result of integrating out conformal free matter fields. The possibilities for the Starobinsky mechanism to produce successful inflation were studied

by Starobinsky himself and by Vilenkin in [3,4]. More recently it was realized by the authors [5] that the standard Einstein action supplemented with a six derivative term, gives rise to a modified equation of motion supporting de Sitter solutions. Moreover, the introduction of this term is not arbitrary at all but it is necessary for the renormalizability of the effective low-energy theory of gravitation at the two-loop level [6].

Whatever could be the most fundamental theory of gravitation (like superstrings or any other) it is clear that at low energy it must lead to the standard Einstein lagrangian proportional to the scalar curvature R which has two derivatives of the metric $g_{\alpha\beta}$ and therefore leads to graviton scattering amplitudes of the order of the external momenta over the Plank mass M_P squared.

This is a good approximation at low energies but when higher energies are considered, higher derivative terms will in general appear in the effective gravitational lagrangian. These terms will be affected by adimensional constants and the necessary M_P factors depending on the dimension of the operator. The adimensional constants play a double role: First they carry the information about the underlying theory of gravitation, second they will absorb the divergences that appear when quantum corrections i.e. loops are computed with the effective lagrangian. These loops can contain matter fields, like in the Starobinsky case, or gravitons (together with the corresponding ghosts) as it happens in the model considered in [5] or whatever. This scheme has many analogies with the phenomenological lagrangian ap-

proach proposed by Weinberg [7] for the description of the low-energy hadron interactions and further elaborated by Gasser and Leutwyler [8]. This technique has also been applied to the parametrization of the scattering amplitudes of the longitudinal components of the electroweak bosons at the TeV scale [9]. More recently [5,10] the same philosophy has been proposed to describe the low-energy action for gravity [11].

At this point a natural question arises: Which are the new higher derivative terms that one must include in the low-energy effective pure gravity lagrangian? In principle, any general covariant combination of scalar curvature and the Ricci and Riemann tensors R , $R_{\alpha\beta}$ and $R_{\alpha\beta\gamma\delta}$ should be included. For example one has three possible independent four derivative terms that can be written as R^2 , $R^{\alpha\beta}R_{\alpha\beta}$ and $-4R^{\alpha\beta}_{\gamma\delta}R^{\gamma\delta}_{\alpha\beta} + 16R^{\alpha\beta}R_{\alpha\beta} - 4R^2$. The third term is a total derivative related with the Euler class of the space-time manifold.

In the general case we will assume the action for gravity to be the space-time integral of an arbitrary local analytical scalar function of the scalar curvature and the Ricci and Riemann tensors added to the standard Hilbert-Einstein action or, in other words:

$$S_G = \int d^n x \sqrt{-g} \left(-\frac{R + \lambda}{16\pi\bar{G}} + F(R, R_{\alpha\beta}, R_{\alpha\beta\gamma\delta}) \right) \quad (1)$$

where \bar{G} denotes the gravitational constant in n dimensions. Note however that in principle the effective action for gravity should contain also non-local terms but these will not be considered here.

As it was discussed above, in this work we are interested in studying the possibilities of having inflation without the introduction of any inflaton field. Therefore we have to deal with the problem of the determination of the precise conditions to be required on the function F appearing in the action in eq.1 leading to de Sitter space-times as solutions of the corresponding equation of motion.

2 Existence of (anti-)de Sitter solutions

For maximally symmetric space-times it is always possible to write the Riemann tensor just in terms of the scalar curvature:

$$R_{\alpha\beta\gamma\delta} = \frac{1}{n(n-1)}R(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) \quad (2)$$

where n is the dimension of space-time. So, we can rewrite the action integral in eq.1 (in four dimensions without cosmological constant) in terms of the scalar curvature only:

$$S_G = \int d^4x \sqrt{-g} \left(-\frac{M_p^2}{16\pi} R + G(R) \right) \quad (3)$$

for some well defined analytical function G to be obtained from the original F function in eq.1. We will first do the study in four dimensions but at the end of this section we will consider also more general scenarios.

The equation of motion corresponding to this action can be found to be:

$$\frac{M_p^2}{16\pi}(R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R) = -\frac{1}{2}g_{\alpha\beta}G(R) + R_{\alpha\beta}G'(R) + g_{\alpha\beta}\square G'(R) - G'(R)_{;\alpha\beta} \quad (4)$$

where a prime denotes derivative respect to R . In order to find the condition for having inflationary space-times as solution of this equation it is important to remember that the (anti-)de Sitter space-time is a maximally symmetric space of constant scalar curvature R . Therefore, for this case the above equation of motion reduces to:

$$\frac{M_p^2}{16\pi}R = 2G(R) - RG'(R) \quad (5)$$

which is the condition for having (anti-)de Sitter solutions. In principle, this equation can have zero, one or several solutions. In the first case no inflation is possible but in the other cases one or more inflationary phases can be present. It is immediate to see from the equation above that the addition of quadratic terms to the Einstein-Hilbert action (i.e. $F = \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, or in other words $G(R) \propto R^2$) does not drive to any solutions different from $R = 0$ which corresponds to the Minkowsky space.

Now let us extend this analysis to arbitrary dimensions. The interest in studying higher derivative gravitational actions in higher dimensions arises from the fact that they could provide a mechanism for obtaining the compactification of the extra dimensions in theories like Kaluza-Klein or superstrings [12],[13], on the other hand it has been also shown that this kind of actions drive to modified Einstein equations for the ordinary four dimensional

space-time supporting inflationary solutions [14].

Following the previous study in four dimensions, we will consider now the $n = 4 + D$ dimensional action in eq.1 and we will concentrate in studying the conditions on function F that lead to a de Sitter geometry for the four dimensional space-time and to spontaneous compactification of the extra dimensions, or in other words, we search under which conditions the equations of motion derived from this action admit a solution corresponding to a product space-time $M \times K$ with M a maximally symmetric four dimensional space-time and for simplicity we will assume K to be a compact D-sphere S^D . Therefore we can write the $4 + D$ dimensional metric tensor as follows:

$$d\tau^2 = -g_{\mu\nu}^M dx^\mu dx^\nu - g_{ij}^K dy^i dy^j \quad (6)$$

with $g_{ij}^K dy^i dy^j = b^2(t) d\Omega_D^2$ and where we have splitted the coordinates on $M \times K$ as $X = (x^\mu, y^j)$, x^μ with $\mu = 1, \dots, 4$ space-time coordinates, y^j with $j = 5, \dots, 4 + D$ internal coordinates and $d\Omega_D^2$ is the metric tensor on a unit D-sphere. It is important to notice that the temporal dependence of $b(t)$ would appear in the four dimensional effective action as a temporal dependence of the physical constants (G , λ and the coefficients of the higher order terms), since strong limits [15] have been imposed to the range of variability of fundamental constants throughout the evolution of the Universe, it is in principle quite accurate to make $\dot{b} = 0$. Taking this into account, any scalar contraction of a $4 + D$ dimensional tensor can be written as a

sum of the corresponding four and D dimensional contractions, for instance:

$R^{\mu\nu}R_{\mu\nu}|_{M \times K} = R^{\mu\nu}R_{\mu\nu}|_M + R^{\mu\nu}R_{\mu\nu}|_K$, this fact together with eq.2 enables to write the action integral as follows:

$$S = \int d^4x d^Dy \sqrt{-g_M} \sqrt{-g_K} \left(-\frac{R_M}{16\pi\bar{G}} - \frac{R_K}{16\pi\bar{G}} - \frac{\lambda}{16\pi\bar{G}} + G_4(R_M) + G_D(R_K) + H(R_M, R_K) \right) \quad (7)$$

Here we have formally gathered terms depending only on R_M in function G_4 , terms depending on R_K in G_D , crossed terms are included in $H(R_M, R_K)$ and we assume without loosing generality $G_D(0) = G_4(0) = H(0, R_K) = H(R_M, 0) = 0$.

The equations of motion corresponding to this action can be found in a similar fashion to eq.4 and using the fact that M and K are both spaces of constant curvature they drive to:

$$\begin{aligned} & \frac{1}{16\pi\bar{G}} \left(-\frac{1}{4}R_M - \frac{1}{2}(R_K + \lambda - 16\pi\bar{G}G_D(R_K)) \right) = \\ & -\frac{1}{2}(G_4(R_M) + H(R_M, R_K)) + \frac{R_M}{4}(G'_4(R_M) + \frac{\partial H}{\partial R_M}) \end{aligned} \quad (8)$$

for the ordinary space-time metric components and for the internal components we obtain:

$$\begin{aligned} & \frac{1}{16\pi\bar{G}} \left(\frac{2-D}{2D}R_K - \frac{1}{2}(R_M + \lambda - 16\pi\bar{G}G_4(R_M)) \right) = \\ & -\frac{1}{2}(G_D(R_K) + H(R_M, R_K)) + \frac{R_K}{D}(G'_D(R_K) + \frac{\partial H}{\partial R_K}) \end{aligned} \quad (9)$$

These are the conditions, analogous to eq.5, that the function F should satisfy in order to get the compactification of the internal dimensions provided the ordinary space-time is a de Sitter one with constant scalar curvature R_M . In principle these are rather natural conditions and no fine tuning on F seems to be needed to satisfy them. However we are interested only in solutions of eq.8 and eq.9 leading to solutions of the standard Einstein equations for the external space in the limit of low external curvature. Then, in that limit we obtain the following two conditions on $G_D(R_K)$:

$$G_D(R_K) = \frac{R_K + \lambda}{16\pi\bar{G}} \quad (10)$$

and

$$G'_D(R_K) = \frac{1}{16\pi\bar{G}} \quad (11)$$

It is then possible to conclude that for an arbitrary function $G_D(R_K)$ the above two conditions will not be simultaneously fulfilled in general and certain unnatural choices of the coefficients of F will be needed. This is in contrast with the ordinary case where a Minkowsky $R_M = 0$ solution always exists in eq.5. In particular we have checked that for the higher order term studied in [13]:

$$F = \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \gamma R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \quad (12)$$

the conditions in eq.10 and eq.11 drive to the fine tuning of the cosmological constant found there.

In principle this results strongly depend on the topology of the compactified dimensions, however it is possible to show that, in the case of a static D-torus and a de Sitter external space-time, one is faced with the same problem, namely, the fine tuning of the parameters, specifically the equations of motion drive in this case to:

$$G_4(R_M) = \frac{R_M + \lambda}{16\pi\bar{G}} \quad (13)$$

and

$$G'_4(R_M) = \frac{1}{16\pi\bar{G}} \quad (14)$$

Therefore, everything seems to indicate that, the higher dimensional context, requires the fine tuning of the parameters of the initial action in order to have, at least asymptotically, an internal space of constant size and simultaneously recover the Einstein equations of motion for the external space in the limit of low external curvature. For this reason, in the following we will concentrate only in the four dimensional case.

3 Robertson-Walker perturbations

Let us consider now the standard Robertson-Walker metric:

$$d\tau^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (15)$$

(we will follow the notation convention of Weinberg's book [16]).

where $k = 1, 0, -1$ corresponds to a closed, flat or open space and $a(t)$ is the universe scale parameter. For the sake of simplicity, in the following we will concentrate in the flat or $k = 0$ universe. This is not in general a maximally symmetric space-time but it includes a particular case which indeed is, namely, $a(t) = a(t_0) \exp H_0(t - t_0)$. This fact enables to use this metric to study small perturbations around the (anti-)de Sitter solutions found in eq.5 and, in turn, the perturbative analysis will provide an estimation of the duration of each inflationary period as we will see later on. Proceeding in this way we first note that the Riemann tensor for this metric has only the following six non-vanishing independent components:

$$R^{tr}{}_{tr} = R^{t\theta}{}_{t\theta} = R^{t\phi}{}_{t\phi} = -\frac{\ddot{a}}{a} \quad (16)$$

$$R^{\theta\phi}{}_{\theta\phi} = R^{r\theta}{}_{r\theta} = R^{r\phi}{}_{r\phi} = -\frac{\dot{a}^2}{a^2} \quad (17)$$

Since only this functions of a and its derivatives will appear in the action it is then more useful in practice to work with them as new variables, thus we define: $b(t) = \log(a(t))$, $H(t) \equiv \dot{b}(t) = \dot{a}/a$ and $\dot{H}(t) = \ddot{b}(t) = \ddot{a}/a - \dot{a}^2/a^2$. In terms of the Hubble parameter $H(t)$ that we have just defined, the (anti-)de Sitter space corresponding to $a(t) = a(t_0) \exp H_0(t - t_0)$ is simply $H = H_0$ and the action integral in eq.1 reads:

$$S_G \propto \int dt e^{3b} L(H, \dot{H}) \quad (18)$$

here a global volume factor has been extracted and the function $L(H, \dot{H})$ which is obtained from the function F in eq. 1 includes the Einstein-Hilbert part of the lagrangian. The equation of motion for the Hubble parameter H obtained from the action above reads:

$$3L - 3H \frac{\partial L}{\partial H} - \frac{d}{dt} \frac{\partial L}{\partial \dot{H}} + 3\dot{H} \frac{\partial L}{\partial \dot{H}} + 9H^2 \frac{\partial L}{\partial \ddot{H}} + 6H \frac{d}{dt} \frac{\partial L}{\partial \dot{H}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{H}} = 0 \quad (19)$$

This equation obviously possesses the solutions $H(t) = H_0$ corresponding to the (anti-)de Sitter space found in eq. 5. In order to study the stability of these solutions we must consider the behavior of small perturbation around them, that is:

$$H(t) = H_0 + \delta(t), \quad \dot{H}(t) = \dot{\delta}(t) \quad (20)$$

Expanding $L(H, \dot{H})$ and its derivatives to the first order in δ we find:

$$L = L_0 + L_1 \delta + L_2 \dot{\delta} + O(\delta^2) \quad (21)$$

$$\frac{\partial L}{\partial H} = L_1 + L_{11} \delta + L_{12} \dot{\delta} + O(\delta^2) \quad (22)$$

$$\frac{\partial L}{\partial \dot{H}} = L_2 + L_{21} \delta + L_{22} \dot{\delta} + O(\delta^2) \quad (23)$$

where we have defined the following coefficients:

$$\begin{aligned}
L_0 &= L(H_0, 0) \\
L_1 &= \frac{\partial L}{\partial H}(H_0, 0) \\
L_2 &= \frac{\partial L}{\partial \dot{H}}(H_0, 0) \\
L_{11} &= \frac{\partial^2 L}{\partial H^2}(H_0, 0) \\
L_{12} = L_{21} &= \frac{\partial^2 L}{\partial H \partial \dot{H}}(H_0, 0) \\
L_{22} &= \frac{\partial^2 L}{\partial \dot{H}^2}(H_0, 0)
\end{aligned} \tag{24}$$

Finally, substituting these expansions in eq.19 and neglecting terms of second order we obtain the linearized equation for $\delta(t)$, which can be written in the following way:

$$3(L_0 - H_0 L_1 + 3H_0^2 L_2) + \frac{1}{3H_0} \frac{d}{dt} \mathcal{F} + \mathcal{F} = 0 \tag{25}$$

where \mathcal{F} stands for:

$$\mathcal{F} = L_{22} \ddot{\delta} + 3H_0 L_{22} \dot{\delta} + \delta(6L_2 + 3H_0 L_{12} - L_{11}) \tag{26}$$

Now the condition in eq. 5 for having (anti-)de Sitter solutions can be rewritten as a condition on H_0 :

$$L_0 - H_0 L_1 + 3H_0^2 L_2 = 0 \tag{27}$$

The resolution of the linearized equation for $\delta(t)$ (eq.25) around each solution $H_0^{(i)}$ of eq.27 simply becomes the resolution of equation $\mathcal{F} = 0$ whose solutions are any linear combination of modes $\exp(t/\tau)$ with:

$$\frac{1}{\tau^{(i)}} = -\frac{3H_0^{(i)}}{2} \pm \left(\frac{9H_0^{(i)2}}{4} - \frac{1}{L_{22}^{(i)}} (6L_2^{(i)} + 3H_0^{(i)}L_{12}^{(i)} - L_{11}^{(i)}) \right)^{(1/2)} \quad (28)$$

Here i runs over the number of real solutions of eq.27. The stability of this inflationary solutions is given by the sign of $\tau^{(i)}$, if $\tau^{(i)} < 0$ then the corresponding solution is stable, otherwise it is unstable. From eq.28 a stable mode is always present. The value of $\tau^{(i)}$ for unstable modes gives an estimation of the duration of the corresponding inflationary period and, in turn, of the number of e -folds $N_e^{(i)}$ produced during this period:

$$N_e^{(i)} = \int H(t)dt = H_0^{(i)}\tau^{(i)} \quad (29)$$

With the simple method described above it becomes very easy to decide if some given generalized effective action for gravity gives rise or not to exponential inflationary solutions and, in that case, to obtain an estimation of the change of the scale produced in the corresponding inflationary phase. In the next sections we will apply the method to some different scenarios which have been considered in the literature in different contexts.

4 The two-loop counterterm

The first example we will consider is that of [5]. There the authors studied the minimal consistent effective low energy two-loop renormalizable lagrangian for pure gravity, this lagrangian is shown to contain a six derivative term [6] added to the usual Einstein-Hilbert one:

$$\mathcal{L}_{eff} = -\frac{M_p^2}{16\pi}R + \frac{\alpha}{M_p^2}R^{\alpha\beta}{}_{\delta\gamma}R^{\delta\gamma}{}_{\sigma\rho}R^{\sigma\rho}{}_{\alpha\beta} \quad (30)$$

The function $L(H, \dot{H})$ appearing in eq.18 is obtained just by rewriting eq.30 for the flat Robertson-Walker space:

$$L(H, \dot{H}) = \frac{6M_p^2}{16\pi}(\dot{H} + 2H^2) - 24\frac{\alpha}{M_p^2}((\dot{H} + H^2)^3 + H^6) \quad (31)$$

By solving the eq.27 we find the inflationary periods produced as a consequence of the introduction of this higher order term. The equation reads:

$$\frac{18M_p^2}{16\pi}H_0^2 + \frac{72\alpha}{M_p^2}H_0^6 = 0 \quad (32)$$

with the obvious solutions $H_0 = 0$ and $H_0^4 = -M_p^4/(64\pi\alpha)$. Therefore, there exists only one inflationary period provided α is negative. On the other hand, eq.29 gives the number of e -folds produced before the Universe leaves this stage of exponential growing, it is immediate to obtain: $N_e \simeq 4.81$. It is important to notice that N_e does not depend on the α coefficient preceding it. Therefore, the addition of the six derivative term to the Einstein action

considered here does not seem to produce enough inflation to solve the problems of the standard cosmology.

5 The effect of matter

This model is based on the semiclassical Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G \langle T_{\mu\nu} \rangle \quad (33)$$

Where $\langle T_{\mu\nu} \rangle$ is the vacuum expectation value of the stress tensor of a number of massless conformally invariant quantum fields with different spin values. As it is well known this vacuum expectation value will be divergent in general and some regularization method will be needed to give it a sense. These divergences affecting $\langle T_{\mu\nu} \rangle$ has been computed in the literature and cause the appearance of fourth order operators at the leading adiabatic order (see [17] for a very complete review). The corresponding higher order terms in the matter lagrangian may be considered also as part of the gravitational effective lagrangian since they are pure geometric objects i.e. they depend only on the standard Riemann and Ricci tensor and the curvature scalar, but not on the matter fields themselves. In dimensional regularization and using the fact that $-4R^{\alpha\beta}_{\gamma\delta}R^{\gamma\delta}_{\alpha\beta} + 16R^{\alpha\beta}R_{\alpha\beta} - 4R^2$ is a total divergence in four dimensions, this divergent contribution to the effective lagrangian can be written for massless fields as:

$$\mathcal{L}_{div} = \frac{1}{(4\pi)^2} \left(\frac{1}{\epsilon} - \frac{\gamma_e}{2} \right) a_2(x) \quad (34)$$

with:

$$a_2(x) = \beta_1 R^2 + \gamma R_{\mu\nu} R^{\mu\nu} \quad (35)$$

$$\beta_1 = -\frac{N_{sc}}{180} + \frac{N_{gh}}{90} + \frac{N_\nu}{60} + \frac{N_{fe}}{30} - \frac{N_v}{20} \quad (36)$$

$$\gamma = \frac{N_{sc}}{60} - \frac{N_\nu}{20} - \frac{N_{fe}}{10} + \frac{7N_v}{30} - \frac{N_{gh}}{30} \quad (37)$$

where we have assumed N_{sc} scalars, N_{gh} ghosts, N_ν neutrinos, N_{fe} Dirac fermions and N_v vectors fields to be present.

Now we have to deal with the problem of the divergent coefficient multiplying these terms. In principle, one may estimate in a heuristic way the value of the corresponding renormalized coefficients by performing the substitution $1/\epsilon - \gamma_e/2 \rightarrow \log(\frac{M_p}{M})$ which is equivalent to the assumption of integrating the matter fields modes from some infrared cutoff M to the M_p scale. Thus, the corresponding renormalized effective lagrangian will be:

$$\mathcal{L}_G = -\frac{M_p^2}{16\pi} R + \frac{1}{(4\pi)^2} \log\left(\frac{M_p}{M}\right) a_2(x) \quad (38)$$

On the other hand, the finite remainder $\langle T_{\mu\nu} \rangle_{ren}$ is in general quite difficult to compute. However, in the case of conformally flat spaces and quantum fields which are also conformally invariant $\langle T_{\mu\nu} \rangle_{ren}$ can be exactly computed out of the knowledge of the trace anomaly and it renders:

$$\langle T_{\mu\nu} \rangle_{ren} = \beta_2^{(1)} H_{\mu\nu} + \rho^{(3)} H_{\mu\nu} + {}^{(4)} H_{\mu\nu} \quad (39)$$

where:

$${}^{(1)} H_{\mu\nu} = 2R_{;\mu\nu} - 2g_{\mu\nu} \square R - \frac{1}{2} g_{\mu\nu} R^2 + 2R R_{\mu\nu} \quad (40)$$

$${}^{(3)} H_{\mu\nu} = R_{\mu}{}^{\sigma} R_{\nu\sigma} - \frac{2}{3} R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma\tau} R_{\sigma\tau} + \frac{1}{4} g_{\mu\nu} R^2 \quad (41)$$

${}^{(4)} H_{\mu\nu}$ is a border term and will be omitted. It is possible to derive the first term above ${}^{(1)} H_{\mu\nu}$ from an action integral in this way:

$${}^{(1)} H_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int \sqrt{-g} R^2 d^n x \quad (42)$$

Note that in general ${}^{(3)} H^{\mu\nu}$ cannot be obtained by varying a local action, although several non-local actions have been proposed which drive to this term. In spite of this, in the case we are considering, conformally flat space, a local action can be found [18] which can be written in terms of the Hubble parameter as:

$$\Gamma = \int dt e^{3b} H^4 \quad (43)$$

It is then possible to write a matter lagrangian in the case of conformally flat space which drives to $\langle T_{\mu\nu} \rangle_{ren}$ and in terms of the Hubble parameter takes the form :

$$L_M = \beta_2 (\dot{H} + 2H^2)^2 + \rho H^4 \quad (44)$$

with the coefficients β_2 and ρ given by:

$$\beta_2 = -\frac{1}{144\pi^2} \frac{1}{120} (N_{sc} + 3N_{\nu} + 6N_{fe} - 18N_v - 2N_{gh}) \quad (45)$$

$$\rho = -\frac{1}{8\pi^2} \frac{1}{360} (-N_{sc} - \frac{11}{2}N_\nu - 11N_{fe} - 62N_v + 2N_{gh}) \quad (46)$$

Therefore we can write the function $L(H, \dot{H})$ in eq.18 including the gravitational and matter sectors as:

$$\begin{aligned} L(H, \dot{H}) = L_G + L_M = & \frac{6M_p^2}{16\pi} (\dot{H} + 2H^2) + \rho H^4 \\ & + 36 \left(\frac{1}{(4\pi)^2} \log\left(\frac{M_p}{M}\right) \beta_1 + \beta_2 \right) (\dot{H} + 2H^2)^2 \\ & + 12 \frac{1}{(4\pi)^2} \log\left(\frac{M_p}{M}\right) \gamma (\dot{H}^2 + 3\dot{H}H^2 + 3H^4) \end{aligned} \quad (47)$$

As we did in the previous example we use eq.27 in order to get the inflationary periods and we obtain in this case:

$$-\frac{M_p^2}{8\pi} H_0^2 + \rho H_0^4 = 0 \quad (48)$$

whose solutions are simply $H_0 = 0$ and $H_0^2 = M_p^2/(8\pi\rho)$. Thus we find again only one de Sitter phase provided ρ is positive. Finally eq.29 will provide the number of e-folds during this phase. For an infrared cut-off $M \simeq 100\text{GeV}$ eq.29 yields:

$$\frac{1}{N_e} = -\frac{3}{2} + \sqrt{\frac{9}{4} + \frac{6\rho}{8.92\beta_1 + 2.97\gamma + 36\beta_2}} \quad (49)$$

For the Standard Model of elementary particles interactions based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ we have $N_{sc} = 4$, $N_\nu = 3$, $N_{fe} = 21$, $N_v = 12$ and $N_{gh} = 12$. By substituting for these values in eqs.48,49, one obtains: $H_0 = 1.52M_p$ and $N_e \simeq 44$. In a similar fashion, other models with

additional content of matter fields can be also considered.

6 Conclusions

In this work we have studied the possibility of having a phase of exponential inflation starting from an effective lagrangian for gravity which is an arbitrary function of the Riemann and Ricci tensors and the curvature scalar. Therefore, our main assumption is that there is some epoch in the early history of our universe where its evolution can be described in a classical way but with an unknown effective action for the gravitational field. We have set the precise conditions on this action for such an exponential expansion to take place. We have shown that it is a rather common phenomenon in four dimensions, even without adding a cosmological constant, since for a generic action eq.5 may have solutions different from zero.

However, in higher dimensions the appearance of additional conditions makes it necessary a precise (fine) tuning of some parameters of the action. Therefore, these models do not seem to provide a natural mechanism for achieving inflation with modified gravitational actions.

We have also studied the stability of the inflationary phase in four dimensions and estimate its duration in classical terms. With these formal tools we have considered some particular models giving rise to higher derivative terms for the gravity action.

The first case to which we have applied our general method is the study of the effect of including the six derivative term needed to renormalize two-loop pure quantum gravity. With this new term added to the standard Einstein action we find that an exponential inflationary solution appear. The corresponding inflationary phase produces 4.8 foldings independently of the concrete value of the six derivative term coupling. Thus we can conclude that this new term does not seem to produce inflation enough to solve any of the problems of the standard cosmology.

As a second example we have applied our general method to study the possibility of having inflation driven by the terms induced in the effective low-energy action for gravitation by integrating out the matter fields. In particular we have found the interesting result that the Standard Model matter produce, by itself, an inflationary phase with $N_e \simeq 44$. This is probably not sufficient to solve the flatness and the horizon problem (see [19] for a recent discussion about the precise conditions needed for that) but it is large enough to take seriously the possibility of having an Standard Model driven larger inflation as an output of more detailed computations.

In conclusion we consider that, in absence of a fundamental theory of gravitation, the possibility of having inflation without the somewhat artificial artifact of the inflaton field, should not be ruled out. Moreover, the results found here for the case of the Standard Model matter coupled to classical gravity, seems to suggest that it is worth to study in deep the dynamics of this system that, finally, is the only one that we are sure is realized in Nature.

Work is in progress in this direction.

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References

- [1] A.H. Guth *Phys. Rev.* **D23**, 347 (1981)
- [2] A. D. Linde *Rep. Prog. Phys.* **47**, 925 (1984)
- [3] A.A. Starobinsky, *Phys. Lett.* **91B** 99 (1980)
- [4] A. Vilenkin, *Phys. Rev.* **D32**, 2511 (1985)
- [5] A. Dobado and A. López, *Phys. Lett.* **B316** 250 (1993)
- [6] D.M. Capper, J.J. Dulwich and M. Ramón Medrano, *Nucl. Phys.* **B254** 737 (1985)
M.H. Goroff and A. Sagnotti, *Nucl. Phys.* **B266** 709 (1986)
- [7] S. Weinberg, *Physica* **96A** 327 (1979)
- [8] J. Gasser and H. Leutwyler, *Ann. of Phys.* **158** 142 (1984) , *Nucl. Phys.* **B250** 465 and 517 (1985)
- [9] A. Dobado and M.J. Herrero, *Phys. Lett.* **B228** 495 (1989) and **B233** 505 (1989)
J. Donoghue and C. Ramirez, *Phys. Lett.* **B234** 361 (1990)
- [10] J.F. Donoghue *Phys. Rev. Lett.* **72** 2996 (1994)
- [11] *Effective Action in Quantum Gravity*, I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, IOP Publishing Ltd (1992).

- [12] *Superstring theory* , M.B. Green, J.H. Schwarz and E. Witten.
Cambridge University Press (1987)
- [13] C. Wetterich, *Phys. Lett.* **B113** 377 (1982)
- [14] Q. Shafi, C. Wetterich, *Phys. Lett.* **B129** 387 (1983)
M.C. Bento, O. Bertolami, *Phys. Lett.* **B228** 348 (1989)
- [15] *The Early Universe*, E.W. Kolb & M.S. Turner, Addison-Wesley
(1990)
- [16] *Gravitation and Cosmology*, S. Weinberg, John Wiley & Sons
(1972)
- [17] *Quantum fields in curved space*, N.D. Birrell and P.C.W. Davies,
Cambridge University Press (1982)
- [18] M.V. Fischetti, J.B. Hartle and B.L. Hu *Phys. Rev.* **D20**, 1757
(1979)
- [19] Y. Hu, M.S. Turner, E.J. Weinberg, *Phys. Rev.* **D49**, 3830 (1994)